

LTH-779
 BU-HEPP 07-07
 CASPER 07-03
 hep-th/

Quasi-realistic heterotic-string models with vanishing one-loop cosmological constant and perturbatively broken supersymmetry?

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Abstract

Quasi-realistic string models in the free fermionic formulation typically contain an anomalous $U(1)$, which gives rise to a Fayet-Iliopoulos D -term that breaks supersymmetry at the one-loop level in string perturbation theory. Supersymmetry is traditionally restored by imposing F - and D -flatness on the vacuum. By employing the standard analysis of flat directions we present a quasi-realistic three generation string model in which stringent F - and D -flat solution do not appear to exist to all orders in the superpotential. We speculate that this result is indicative of the non-existence of supersymmetric flat F - and D -solutions in this model. We provide some arguments in support of this scenario and discuss its potential implications. Bose-Fermi degeneracy of the string spectrum implies that the one-loop partition function and hence the one-loop cosmological constant vanishes in the model. If our assertion is correct, this model may represent the first known example with vanishing cosmological constant and perturbatively broken supersymmetry. We discuss the distinctive properties of the internal free fermion boundary conditions that may correspond to a large set of models that share these properties. The geometrical moduli in this class of models are fixed due to asymmetric boundary conditions, whereas absence of supersymmetric flat directions would imply that the supersymmetric moduli are fixed as well and the dilaton may be fixed by hidden sector nonperturbative effects.

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1 Introduction

The quasi-realistic heterotic-string models in the free fermionic formulation, which are related to $Z_2 \times Z_2$ orbifold compactifications, are among the most realistic string models constructed to date. These models produce a rich variety of three generation models with the canonical $SO(10)$ embedding of the Standard Model spectrum, and include: the flipped $SU(5)$ string models [1] (FSU5); the standard-like string models [2, 3, 4, 5, 6, 7]; the Pati-Salam string models [8] (PS); the Left-Right symmetric string models [9] (LRS). Many of the issues pertaining to the phenomenology of the Standard Model and Grand Unification were investigated in the context of these models [10]. Additionally, the free fermionic models produced the first known string models in which the matter content in the observable Standard Model charged sector of the effective low energy quantum field theory consists solely of the Minimal Supersymmetric Standard Model [5, 6].

A common feature of many of the quasi-realistic free fermionic heterotic-string models is the existence of an “anomalous” $U(1)$ symmetry [11]. The anomalous $U(1)_A$ is broken by the Green-Schwarz-Dine-Seiberg-Witten mechanism [12] in which a potentially large Fayet-Iliopoulos D -term ξ is generated by the VEV of the dilaton field. Such a D -term would, in general, break supersymmetry, unless there is a direction $\hat{\phi} = \sum \alpha_i \phi_i$ in the scalar potential for which $\sum Q_A^i |\alpha_i|^2$ is of opposite sign to ξ and that is D -flat with respect to all the non-anomalous gauge symmetries, as well as F -flat. If such a direction exists, it will acquire a VEV, cancelling the Fayet-Iliopoulos ξ -term, restoring supersymmetry and stabilising the vacuum. The set of D - and F -flat constraints is given by

$$\langle D_A \rangle = \langle D_\alpha \rangle = 0 ; \quad \langle F_i \equiv \frac{\partial W}{\partial \eta_i} \rangle = 0 ; \quad (1.1)$$

$$D_A = \left[K_A + \sum Q_A^k |\chi_k|^2 + \xi \right] ; \quad (1.2)$$

$$D_\alpha = \left[K_\alpha + \sum Q_\alpha^k |\chi_k|^2 \right] , \quad \alpha \neq A ; \quad (1.3)$$

$$\xi = \frac{g^2 (\text{Tr} Q_A)}{192 \pi^2} M_{\text{Pl}}^2 ; \quad (1.4)$$

where χ_k are the fields which acquire VEVs of order $\sqrt{\xi}$, while the K -terms contain fields η_i like squarks, sleptons and Higgs bosons whose VEVs vanish at this scale. Q_A^k and Q_α^k denote the anomalous and non-anomalous charges, and $M_{\text{Pl}} \approx 2 \times 10^{18}$ GeV denotes the reduced Planck mass. The solution (*i.e.* the choice of fields with non-vanishing VEVs) to the set of equations (1.1)–(1.3), though nontrivial, is not unique. Therefore in a typical model there exist a moduli space of solutions to the F and D flatness constraints, which are supersymmetric and degenerate in energy [13]. Much of the study of the superstring models phenomenology (as well as non-string supersymmetric models [14]) involves the analysis and classification of these

flat directions. The methods for this analysis in string models have been systematised in [15, 16, 5, 17].

In general it has been assumed in the past that in a given string model there should exist a supersymmetric solution to the F and D flatness constraints. The simpler type of solutions utilise only fields that are singlets of all the non-Abelian groups in a given model (type I solutions). More involved solutions (type II solutions), that utilise also non-Abelian fields, have also been considered [17], as well as inclusion of non-Abelian fields in systematic methods of analysis [17]. The general expectation that a given model admits a supersymmetric solution arises from analysis of supersymmetric point quantum field theories. In these cases it is known that if supersymmetry is preserved at the classical level, *i.e.* tree-level in perturbation theory, then there exist index theorems that forbid supersymmetry breaking at the perturbative quantum level [18]. Therefore in point quantum field theories supersymmetry breaking may only be induced by non-perturbative effects [19].

Recently we constructed string models [7] in which the issues of supersymmetric flat directions merits further investigation. The aim of ref. [7] was to build models with a reduced untwisted Higgs spectrum. This was achieved in ref. [7] by imposing asymmetric boundary conditions in a boundary condition basis vector that does not break the $SO(10)$ symmetry. An unforeseen consequence of the Higgs reduction mechanism of ref. [7] was the simultaneous projection of untwisted $SO(10)$ singlet fields. Subsequently the moduli space of supersymmetric flat solutions is vastly reduced. In fact, in ref. [7] it was concluded that the model under investigation there does not contain supersymmetric flat directions that do not break some of the Standard Model symmetries. Indeed, for that reason, a phenomenologically viable model with a reduced untwisted Higgs spectrum was not presented in ref. [7].

The question therefore remains whether a phenomenologically viable model with a reduced untwisted Higgs spectrum exists. In this paper we explore this question further. The untwisted Higgs reduction mechanism that we use here differs from the one of ref. [7]. Here we present a model that utilises boundary conditions that are both symmetric and asymmetric in the basis vectors that break $SO(10)$ to $SO(6) \times SO(4)$, with respect to two of the twisted sectors of the $Z_2 \times Z_2$ orbifold. The consequence is that two of the untwisted Higgs multiplets, associated with two of the twisted sectors, are projected entirely from the massless spectrum. As a result, and similar to the model of ref. [7], the string model contains a single pair of untwisted electroweak Higgs doublets.

In the process of seeking such a model with a phenomenologically viable supersymmetric flat direction, we arrive in this paper to the unexpected conclusion that the model may not contain supersymmetric flat directions at all. In the least, this model appears to have no D -flat directions that can be proven to be F -flat to all order, other than through order-by-order analysis. That is, there does not appear to be any D -flat directions with *stringent* F -flatness (as defined in [5, 6, 20]). In the analysis of the flat directions we include all the fields in the string model, *i.e.*

Standard Model singlet states as well as Standard Model charged states. The model therefore does not contain a D -flat directions that is also stringently F -flat to all order of non-renormalizable terms.

The model may of course still admit non-stringent flat directions that rely on cancellations between superpotential terms. However, past experience suggests that non-stringent flat directions can only hold order by order, and are not maintained to all orders [21, 9]. This is the key difference between the string theory case, in which heavy string modes generate an infinite tower of terms, versus the field theory case in which heavy modes are not integrated out. We therefore speculate that in this case supersymmetry is not exact, but is in general broken at some order.

If this finding remains true after the entire parameter space of possible all-order non-stringent flat directions has been examined (an undertaking of several years), we must ask what are the implications. If a model without all-order F -flatness were to be found, then supersymmetry would remain broken by the Fayet–Iliopoulos term at a finite order, which is generated at the one-loop level in string perturbation theory, rather than be cancelled by a D -flat direction with anomalous charge. If so, then this would imply, although supersymmetry is unbroken at the classical level and the string spectrum is Bose–Fermi degenerate, that supersymmetry may be broken at the perturbative quantum level. Nevertheless, since the spectrum is Bose–Fermi degenerate, the one-loop cosmological constant still vanishes.

The string model that we present contains three chiral generations, charged under the Standard Model gauge group and with the canonical $SO(10)$ embedding of the weak–hypercharge; one pair of untwisted electroweak Higgs doublets; a cubic level top–quark Yukawa coupling. The string model therefore shares some of the phenomenological characteristics of the quasi-realistic free fermionic string models. It may therefore represent an example of a quasi-realistic string model, with vanishing one-loop cosmological constant and perturbatively broken supersymmetry.

Our paper is organised as follows: in section 2 we review some aspects of the free fermionic formalism and in section 3 we elaborate on stringent flat directions. Then in section 4 we present the string model. In section 5 we discuss the methodology of flat direction analysis and the evidence for concluding that supersymmetric (stringent) flat directions may not exist to all order in the string model of section 4. Section 6 concludes the paper.

2 Free Fermionic Models

In this section we briefly review the construction and structure of the free fermionic standard like models. The notation and further details of the construction of these models are given elsewhere [2, 3, 22, 5, 9, 7]. In the free fermionic formulation of the heterotic string in four dimensions [23] all the world-sheet degrees of freedom, required to cancel the conformal anomaly, are represented in terms of free fermions propagating on the string world-sheet. In the light-cone gauge the world-sheet

field content consists of two transverse left- and right-moving space-time coordinate bosons, $X_{1,2}^\mu$ and $\bar{X}_{1,2}^\mu$, and their left-moving fermionic superpartners, $\psi_{1,2}^\mu$, and additional 62 purely internal Majorana-Weyl fermions, of which 18 are left-moving and 44 are right-moving. The models are constructed by specifying the phases picked by the world-sheet fermions when transported along the torus non-contractible loops

$$f \rightarrow -e^{i\pi\alpha(f)} f, \quad \alpha(f) \in (-1, 1]. \quad (2.1)$$

Each model corresponds to a particular choice of fermion phases consistent with modular invariance and is generated by a set of basis vectors describing the transformation properties of the 64 world-sheet fermions. The physical spectrum is obtained by applying the generalised GSO projections. The low energy effective field theory is obtained by S-matrix elements between external states [24].

The boundary condition basis defining a typical realistic free fermionic heterotic string model is constructed in two stages. The first stage consists of the NAHE set, which is a set of five boundary condition basis vectors, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [25, 22]. The gauge group, after imposing the GSO projections induced by the NAHE set, is $SO(10) \times SO(6)^3 \times E_8$, with $N = 1$ supersymmetry. The NAHE set divides the internal world-sheet fermions in the following way: $\bar{\phi}^{1,\dots,8}$ generate the hidden E_8 gauge group, $\bar{\psi}^{1,\dots,5}$ generate the $SO(10)$ gauge group, while $\{\bar{y}^{3,\dots,6}, \bar{\eta}^1\}$, $\{\bar{y}^1, \bar{y}^2, \bar{\omega}^5, \bar{\omega}^6, \bar{\eta}^2\}$ and $\{\bar{\omega}^{1,\dots,4}, \bar{\eta}^3\}$ generate the three horizontal $SO(6)$ symmetries. The left-moving $\{y, \omega\}$ states are divided to $\{y^{3,\dots,6}\}$, $\{y^1, y^2, \omega^5, \omega^6\}$, $\{\omega^{1,\dots,4}\}$, while χ^{12} , χ^{34} , χ^{56} generate the left-moving $N = 2$ world-sheet supersymmetry.

The second stage of the basis construction consists of adding to the NAHE set three additional boundary condition basis vectors. These additional basis vectors reduce the number of generations to three chiral generations, one from each of the sectors b_1 , b_2 and b_3 , and simultaneously break the four dimensional gauge group. The assignment of boundary conditions to $\{\bar{\psi}^{1,\dots,5}\}$ breaks $SO(10)$ to one of its subgroups. Similarly, the hidden E_8 symmetry is broken to one of its subgroups. The flavour $SO(6)^3$ symmetries in the NAHE-based models are always broken to flavour $U(1)$ symmetries, as the breaking of these symmetries is correlated with the number of chiral generations. Three such $U(1)_j$ symmetries are always obtained in the NAHE based free fermionic models from the subgroup of the observable E_8 , which is orthogonal to $SO(10)$. These are produced by the world-sheet currents $\bar{\eta}^j \bar{\eta}^{j*}$ ($j = 1, 2, 3$), which are part of the Cartan sub-algebra of the observable E_8 . Additional unbroken $U(1)$ symmetries, denoted typically by $U(1)_j$ ($j = 4, 5, \dots$), arise by pairing two real fermions from the sets $\{\bar{y}^{3,\dots,6}\}$, $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\bar{\omega}^{1,\dots,4}\}$. The final observable gauge group depends on the number of such pairings. Alternatively, a left-moving real fermion from the sets $\{y^{3,\dots,6}\}$, $\{y^{1,2}, \omega^{5,6}\}$ and $\{\omega^{1,\dots,4}\}$ may be paired with its respective right-moving real fermion to form an Ising model operator, in which case the rank of the right-moving gauge group is reduced by one. The reduction of untwisted electroweak Higgs doublets crucially depends on the pairings of the left- and right-moving fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}^{1\dots 6}$.

Subsequent to constructing the basis vectors and extracting the massless spectrum, the analysis of the free fermionic models proceeds by calculating the superpotential. The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \quad (2.2)$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators, using the rules given in [24]. Typically, one of the $U(1)$ factors in the free-fermion models is anomalous and generates a Fayet–Iliopoulos term which breaks supersymmetry at the Planck scale [12]. A supersymmetric vacuum is obtained by assigning non-trivial VEVs to a set of Standard Model singlet fields in the massless string spectrum along F and D -flat directions. Some of these fields will appear in the nonrenormalizable terms (2.2), leading to effective operators of lower dimension. Their coefficients contain factors of order $\mathcal{V}/M \sim 1/10$.

3 Stringent flat directions

In general, systematic analysis of simultaneously D - and F -flat directions in anomalous models is a complicated, non-linear process¹. In weakly coupled heterotic string (WCHS) model-building, F -flatness of a specific VEV direction in the low energy effective field theory may be proven to a given order by cancellation of F -term components, only to be lost a mere one order higher at which cancellation is not found. An exception is directions with stringent F -flatness [5, 6, 20]. Rather than allowing cancellation between two or more components in an F -term, stringent F -flatness requires that each possible component in an F -term have zero vacuum expectation value.

When only non-Abelian singlet fields acquire VEVs, stringent flatness implies that two or more singlet fields in a given F -term cannot take on VEVs. For example, in

¹In ref. [26] it is argued that, in addition to flat directions, isolated special points generically exist in the VEV parameter space that are not located along flat directions, but for which all D - and F -terms are nonetheless zero. Ref. [26] calls upon the proof by Wess and Bagger [27] that non-anomalous D -terms do not actually increase the number of constraints for supersymmetric flat directions beyond the F -term constraints. However, FI term cancellation, which requires the existence of one monomial that is D -flat for all non-anomalous symmetries, but that carries the opposite sign to the FI-term in the anomalous $U(1)_A$, imposes an additional constraint. Thus, without an anomalous $U(1)$, the system of D - and F -equations is not over constraining. In the latter case, once a solution to $F_m = 0$, for all fields s_m^o , is found (to a given order), complexified gauge transformations of the fields s_m^o , that continue to provide a $F_m = 0$ solution, can be performed that simultaneously arrange for non-anomalous D -flatness. Thus, since the $F_m = 0$ equations impose m (non-linear) constraints on m fields, there should be at least one non-trivial non-anomalous D -flat solution for any set of fields s_m^o . A parallel proof for non-Abelian field VEVs also exists. (Complications to these proofs do arise when different scalar fields possess the same gauge charges.) This reduction in apparent total constraints is possible because the F -term equations constrain gauge invariant polynomials, which also correspond to non-anomalous D -flat directions [26].

section 4.1, which presents the third through fifth order superpotential for the model under consideration, the components of the F -term for Φ_{45} are (through third order):

$$F_{\Phi_{45}} = \bar{\Phi}_{46}\bar{\Phi}'_{56} + \bar{\Phi}'_{46}\bar{\Phi}_{56}. \quad (3.1)$$

For stringent F -flatness we require not just that $\langle F_{\Phi_{45}} \rangle = 0$, but that each component within is zero, i.e.,

$$\langle \bar{\Phi}_{46}\bar{\Phi}'_{56} \rangle = 0, \langle \bar{\Phi}'_{46}\bar{\Phi}_{56} \rangle = 0. \quad (3.2)$$

Thus, by not allowing cancellation between components in a given F -term, stringent F -flatness imposes stronger constraints than generic F -flatness, but requires significantly less fine-tuning between the VEVs of fields.

The net effect of all stringent F -constraints on a given superpotential term is that at least two fields in the term must not take on VEVs. This condition can be relaxed when non-Abelian fields acquire VEVs. Self-cancellation of a single component in a given F -term is possible between various VEVs within a given non-Abelian representation. Self-cancellation was discussed in [6] for $SU(2)$ and $SO(2n)$ states.

A given set of stringent F flatness constraints are not independent and solutions to a set can be expressed in the language of Boolean algebra (logic) and applied as constraints to linear combinations of D -flat basis directions. The Boolean algebra language makes clear that the effect of stringent F -flat constraints is strongest for low order superpotential terms and lessens with increasing order. In particular, for the model presented herein, stringent flatness is extremely constraining on VEVs of the reduced number of (untwisted) singlet fields appearing in the third through fifth order superpotential, in comparison to its constraints on the larger number of singlets in the model of [7].

One might imagine that stringent F -flatness constraints requires order-by-order testing of superpotential terms. This is, in fact, not necessary. All-order stringent F -flatness can actually be proven or disproven by examining only a small finite set of possible dangerous (i.e., F -flatness breaking) superpotential terms. Through a process such as matrix singular value decomposition (SVD)², a finite set of superpotential terms can be constructed that generates all possible dangerous superpotential terms for a specific D -flat direction. This basis of gauge-invariants can always be formed with particular attributes: (1) each basis element term contains at most one unVEVed field (since to threaten F -flatness, a gauge-invariant term, necessarily without anomalous charge, can contain no more than one unVEVed field); (2) there is at most one basis term for each unVEVed field in the model; and (3) when an unVEVed field appears in a basis term, it appears only to the first power. The SVD process generated a possibly threading basis of superpotential terms for several models (see for example [5, 6, 9, 17, 29, 30, 31]).

²A SVD fortran subroutine is provided in [28].

To appear in a string-based superpotential, a gauge invariant term must also follow Ramond-Neveu-Schwarz worldsheet charge conservation rules. For free fermionic models these rules were generalized from finite order in [24, 32] to all-order in [17]. The generic all order rules can be applied to systematically determine if any product of SVD-generated F -flatness threatening superpotential basis elements survive in the corresponding string-generated superpotential. If none survive, then F -flatness is proven to all finite order. This technique has been used to prove F -flatness to all finite order for various directions in several models [5, 6, 9, 17, 29, 30, 31]. Alternately, if any terms do survive, the lowest order is determined at which stringent F -flatness is broken.

How should stringent (especially all-order) flat directions be interpreted in comparison to general (perhaps finite order) flat directions? All-order stringent flat directions contain a minimum number of VEVs and appear in models as the roots of more fine-tuned (generally finite-order) flat directions that require specific cancellations between F -term components. The latter may involve cancellations between sets of components of different orders in the superpotential.

All-order stringent flat directions have indeed been discovered to be such roots in all prior free fermionic heterotic models for which we have performed systematic flat direction classifications. However, the model presented herein appears to lack any stringent flat directions, at least within the expected range of VEV parameter space. We have reached this conclusion after employing our standard systematic methodology for D - and F -flat direction analysis.

4 The String Model

In this section we give the details of our model. The boundary condition basis vectors beyond the NAHE-set and the one-loop GSO projection coefficients are shown in eq. (4.1) and eq. (4.2), respectively.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
α	0	0	0	0	1 1 1 0 0	1	0	0	1 1 0 0 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	1	0	0 0 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 0 $\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$

	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	$\bar{y}^1 \bar{\omega}^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$\bar{\omega}^2 \bar{\omega}^4$
α	1	0	0	1	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	1	0	1	0	1
γ	0	1	0	0	0	1	0	0	1	0	0	0

(4.1)

With the choice of generalised GSO coefficients:

$$\begin{array}{c}
\mathbf{1} \quad S \quad b_1 \quad b_2 \quad b_3 \quad \alpha \quad \beta \quad \gamma \\
\mathbf{1} \left(\begin{array}{ccccccccc}
1 & 1 & -1 & -1 & -1 & -1 & -1 & i \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & 1 & i \\
-1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 \\
-1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\
-1 & -1 & 1 & 1 & -1 & -1 & -1 & -i
\end{array} \right)
\end{array} \quad (4.2)$$

In matrix (4.2) only the entries above the diagonal are independent, while those below and on the diagonal are fixed by the modular invariance constraints. Blank lines are inserted to emphasise the division of the free phases between the different sectors of the realistic free fermionic models. Thus, the first two lines involve only the GSO phases of $c\left(\begin{smallmatrix} \mathbf{1}, S \\ a_i \end{smallmatrix}\right)$. The set $\{\mathbf{1}, S\}$ generates the $N = 4$ model with S being the space-time supersymmetry generator and therefore the phases $c\left(\begin{smallmatrix} S \\ a_i \end{smallmatrix}\right)$ are those that control the space-time supersymmetry in the superstring models. Similarly, in the free fermionic models, sectors with periodic and anti-periodic boundary conditions, of the form of b_i , produce the chiral generations. The phases $c\left(\begin{smallmatrix} b_i \\ b_j \end{smallmatrix}\right)$ determine the chirality of the states from these sectors.

Both the basis vectors α and β break the $SO(10)$ symmetry to $SO(6) \times SO(4)$ and the basis vector γ breaks it further to $SU(3) \times U(1)_C \times SU(2) \times U(1)_L$. The basis vector α is symmetric with respect to the sector b_1 and asymmetric with respect to the sectors b_2 and b_3 , whereas the basis vector β is symmetric with respect to b_2 and asymmetric with respect to b_1 and b_3 . As a consequence of these assignments and of the string doublet-triplet splitting mechanism [33], both the untwisted Higgs colour triplets and electroweak doublets, with leading coupling to the matter states from the sectors b_1 and b_2 , are projected out by the generalised GSO projections. At the same time the untwisted colour Higgs triplets that couple at leading order to the states from the sector b_3 are projected out, whereas the untwisted electroweak Higgs doublets remain in the massless spectrum. Due to the asymmetric boundary conditions in the sector γ with respect to the sector b_3 , the leading Yukawa coupling is that of the up-type quark from the sector b_3 to the untwisted electroweak Higgs doublet [4]. Hence, the leading Yukawa term is that of the top quark and only its mass is characterised by the electroweak VEV [4]. The lighter quarks and leptons couple to the light Higgs doublet through higher order nonrenormalizable operators that become effective renormalizable operators by the VEVs that are used to cancel the anomalous $U(1)_A$ D -term equation [4]. The novelty in the construction of ref. [7], and in the model of eq. (4.1), is that the reduction of the untwisted Higgs spectrum is

obtained by the choice of the boundary condition basis vectors in eq. (4.1), whereas in previous models it was obtained by the choice of flat directions and analysis of the superpotential [10].

The final gauge group of the string model arises as follows: in the observable sector the NS boundary conditions produce gauge group generators for

$$SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L \times U(1)_{1,2,3} \times U(1)_{4,5,6} \quad . \quad (4.3)$$

Thus, the $SO(10)$ symmetry is broken to $SU(3) \times SU(2)_L \times U(1)_C \times U(1)_L$, where,

$$U(1)_C = \text{Tr } U(3)_C \Rightarrow Q_C = \sum_{i=1}^3 Q(\bar{\psi}^i) , \quad (4.4)$$

$$U(1)_L = \text{Tr } U(2)_L \Rightarrow Q_L = \sum_{i=4}^5 Q(\bar{\psi}^i) . \quad (4.5)$$

The flavour $SO(6)^3$ symmetries are broken to $U(1)^{3+n}$ with $(n = 0, \dots, 6)$. The first three, denoted by $U(1)_j$ ($j = 1, 2, 3$), arise from the world-sheet currents $\bar{\eta}^j \eta^{j*}$. These three $U(1)$ symmetries are present in all the three generation free fermionic models which use the NAHE set. Additional horizontal $U(1)$ symmetries, denoted by $U(1)_j$ ($j = 4, 5, \dots$), arise by pairing two real fermions from the sets $\{\bar{y}^{3,\dots,6}\}$, $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\bar{\omega}^{1,\dots,4}\}$. The final observable gauge group depends on the number of such pairings. In this model there are the pairings $\bar{y}^3 \bar{y}^6$, $\bar{y}^1 \bar{\omega}^5$ and $\bar{\omega}^2 \bar{\omega}^4$, which generate three additional $U(1)$ symmetries, denoted by $U(1)_{4,5,6}$.

It is important to note that the existence of these three additional $U(1)$ currents is correlated with the assignment of asymmetric boundary conditions with respect to the set of internal world-sheet fermions $\{y, \omega | \bar{y}, \bar{\omega}\}^{1,\dots,6}$, in the basis vectors that extend the NAHE-set, $\{\alpha, \beta, \gamma\}$. This assignment of asymmetric boundary conditions in the basis vector that breaks the $SO(10)$ symmetry to $SO(6) \times SO(4)$ results in the projection of the untwisted Higgs colour-triplet fields and preservation of the corresponding electroweak-doublet Higgs representations [33].

In the hidden sector, which arises from the complex world-sheet fermions $\bar{\phi}^{1\dots 8}$, the NS boundary conditions produce the generators of

$$SU(2)_{1,2,3,4} \times SU(4)_{H_1} \times U(1)_{H_1} . \quad (4.6)$$

$U(1)_{H_1}$ corresponds to the combinations of the world-sheet charges

$$Q_{H_1} = \sum_{i=5}^8 Q(\bar{\phi}^i) . \quad (4.7)$$

The model contains several additional sectors that may a priori produce space-time vector bosons and enhance the gauge symmetry, which include the sectors $\zeta \equiv \mathbf{1} + b_1 + b_2 + b_3$ and $\mathbf{1} + S + \alpha + \beta + \gamma$. Additional space-time vector bosons from

these sectors would enhance the gauge symmetry that arise from the space-time vector bosons produced in the Neveu–Schwarz sector. However, with the choice of generalised GSO projection coefficients given in eq. (4.2) all of the extra gauge bosons from these sectors are projected out and the four dimensional gauge group is given by eqs. (4.3) and (4.6).

In addition to the graviton, dilaton, antisymmetric sector and spin-1 gauge bosons, the Neveu–Schwarz sector gives one pair of electroweak Higgs doublets h_3 and \bar{h}_3 ; six pairs of $SO(10)$ singlets, which are charged with respect to $U(1)_{4,5,6}$; three singlets of the entire four dimensional gauge group. A notable difference as compared to models with unreduced untwisted Higgs spectrum, like the model of ref. [3], is that the $SO(10)$ singlet fields, which are charged under $U(1)_{1,2,3}$, are projected out from the massless spectrum. The three generations are obtained from the sectors b_1 , b_2 and b_3 . The model contain states that are vector-like with respect to the Standard Model and all non-Abelian group factors, but may be chiral with respect to the $U(1)$ symmetries that are orthogonal to the $SO(10)$ group. The full massless spectrum of the model is detailed in Table 1 at the end of this paper.

As a final note we remark that the boundary conditions with respect to the internal world-sheet fermions of the set $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\dots,6}$ in the basis vectors α , β and γ , that extend the NAHE-set, are similar to those in the basis vectors that generate the string model of ref. [3], with the replacements

$$\begin{aligned}\alpha(\bar{y}^3\bar{y}^6) &\longleftrightarrow \gamma(\bar{y}^3\bar{y}^6) \\ \beta(\bar{y}^1\bar{\omega}^5) &\longleftrightarrow \gamma(\bar{y}^1\bar{\omega}^5).\end{aligned}\tag{4.8}$$

The world-sheet fermions $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\dots,6}$ correspond to the compactified dimensions in a corresponding bosonic formulation. The substitutions in (4.8) are augmented with suitable modifications of the boundary conditions of the world-sheet fermions $\{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,\dots,3}, \bar{\phi}^{1,\dots,8}\}$, which correspond to the gauge degrees of freedom. The effect of these additional modifications is to alter the hidden sector gauge group. While the substitutions in (4.8) look innocuous enough, they in fact produce substantial changes in the massless spectrum and, as a consequence, in the physical characteristics of the models. With regard to the flat directions of the superpotential, the effect of these changes on the untwisted states will be particularly noted.

4.1 Third through Fifth Order Superpotential

The three singlets of the entire four dimensional gauge group are obtained from:

$$\begin{aligned}\xi_1 &= \chi^{12*}\bar{\omega}^3\bar{\omega}^6|0> \ , \\ \xi_2 &= \chi^{34*}\bar{\omega}^1\bar{y}^5|0> \ , \\ \xi_3 &= \chi^{56*}\bar{y}^2\bar{y}^4|0> \ .\end{aligned}$$

We show below the cubic through quintic order superpotential terms.

Trilinear superpotential:

$$\begin{aligned}
W_3 = & N_3^c L_3 \bar{h} + u_3^c Q_3 \bar{h} + H_4 \bar{H}_7 h + \bar{H}_4 H_7 \bar{h} + \\
& + \xi_1 (H_1 \bar{H}_1 + H_8 \bar{H}_8 + H_9 \bar{H}_9) \\
& + \xi_2 (H_2 \bar{H}_2 + H_{10} \bar{H}_{10} + H_{11} \bar{H}_{11}) \\
& + \xi_3 (H_3 \bar{H}_3 + H_4 \bar{H}_4 + H_5 \bar{H}_5 + H_6 \bar{H}_6 + H_7 \bar{H}_7) \\
& + \xi_3 (\Phi_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} + \Phi_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta}) \\
& + \Phi_{45} (\bar{\Phi}_{46} \bar{\Phi}'_{56} + \bar{\Phi}'_{46} \bar{\Phi}_{56}) + \bar{\Phi}_{45} (\Phi_{46} \Phi'_{56} + \Phi'_{46} \Phi_{56}) \\
& + \Phi'_{45} (\bar{\Phi}_{46} \Phi_{56} + \bar{\Phi}'_{46} \Phi'_{56}) + \bar{\Phi}'_{45} (\Phi_{46} \bar{\Phi}_{56} + \Phi'_{46} \bar{\Phi}'_{56}) \\
& + \Phi'_{45} ((\Phi_1^{\alpha\beta})^2 + (\Phi_2^{\alpha\beta})^2) + \bar{\Phi}'_{45} ((\bar{\Phi}_1^{\alpha\beta})^2 + (\bar{\Phi}_2^{\alpha\beta})^2) \\
& + \bar{\Phi}'_{45} H_{12} H_{13} + \Phi_{46} H_{14} H_{15} + \bar{\Phi}'_{56} H_{16} H_{17} \\
& + \Phi'_{56} (H_1)^2 + \bar{\Phi}'_{56} (\bar{H}_1)^2 + \bar{\Phi}'_{46} (H_2)^2 + \Phi'_{46} (\bar{H}_2)^2 \\
& + \Phi_1^{\alpha\beta} H_9 H_{11} + \bar{\Phi}_2^{\alpha\beta} (\bar{H}_1 \bar{H}_2 + \bar{H}_8 \bar{H}_{10}) + \bar{H}_1 \bar{H}_4 H_{10} + H_2 \bar{H}_4 \bar{H}_8 \quad (4.9)
\end{aligned}$$

Quartic superpotential:

$$W_4 = Q_1 u_1 H_4 \bar{H}_5 + Q_2 u_2 H_4 \bar{H}_6 + L_1 N_1^c H_4 \bar{H}_5 + L_2 N_2^c H_4 \bar{H}_6 \quad (4.10)$$

Quintic superpotential:

$$\begin{aligned}
W_5 = & Q_1 H_3 L_1 \bar{H}_5 \xi_2 + Q_2 H_3 L_2 \bar{H}_6 \xi_1 + Q_3 u_3^c \bar{H}_1 \bar{H}_7 H_{10} + Q_3 u_3^c H_2 \bar{H}_7 \bar{H}_8 \\
& + d_1^c u_1^c H_3 \bar{H}_5 \xi_2 + d_1^c H_3 H_3 \Phi_{46} V_2 + d_2^c u_2^c H_3 \bar{H}_6 \xi_1 + d_2^c H_3 H_3 \bar{\Phi}'_{56} V_5 \\
& + H_3 \bar{H}_4 \bar{H}_1 \bar{H}_3 H_{10} + H_3 \bar{H}_4 H_2 \bar{H}_3 \bar{H}_8 + H_3 \bar{H}_1 \bar{H}_2 \bar{H}_3 \bar{\Phi}_2^{\alpha\beta} + H_3 \bar{H}_3 \Phi_1^{\alpha\beta} H_{11} H_9 \\
& + H_3 \bar{H}_3 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + L_3 \bar{H}_1 N_3^c \bar{H}_7 H_{10} + L_3 H_2 N_3^c \bar{H}_7 \bar{H}_8 + H_4 H_4 \bar{\Phi}'_{46} H_8 H_8 \\
& + H_4 H_4 \Phi_{46} N_1^c V_2 + H_4 H_4 \bar{\Phi}'_{56} N_2^c V_5 + H_4 H_4 \bar{\Phi}'_{56} \bar{H}_{10} \bar{H}_{10} + H_4 \bar{H}_4 \bar{H}_4 \bar{H}_1 H_{10} \\
& + H_4 \bar{H}_4 \bar{H}_4 H_2 \bar{H}_8 + H_4 \bar{H}_4 \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} + H_4 \bar{H}_4 \Phi_1^{\alpha\beta} H_{11} H_9 + H_4 \bar{H}_4 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} \\
& + H_4 H_1 \xi_2 H_{10} H_8 + H_4 H_2 \bar{\Phi}'_{56} \bar{\Phi}_2^{\alpha\beta} \bar{H}_{10} + \bar{H}_4 \bar{H}_4 \bar{\Phi}'_{46} \bar{H}_8 \bar{H}_8 + \bar{H}_4 \bar{H}_4 \bar{\Phi}'_{56} H_{10} H_{10} \\
& + \bar{H}_4 \bar{H}_1 \bar{H}_1 H_1 H_{10} + \bar{H}_4 \bar{H}_1 H_1 H_2 \bar{H}_8 + \bar{H}_4 \bar{H}_1 \bar{H}_2 H_2 H_{10} + \bar{H}_4 \bar{H}_1 H_7 \bar{H}_7 H_{10} \\
& + \bar{H}_4 \bar{H}_1 H_6 \bar{H}_6 H_{10} + \bar{H}_4 \bar{H}_1 H_5 \bar{H}_5 H_{10} + \bar{H}_4 \bar{H}_1 \xi_2 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 + \bar{H}_4 \bar{H}_1 \Phi_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} H_{10} \\
& + \bar{H}_4 \bar{H}_1 \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} H_{10} + \bar{H}_4 \bar{H}_1 H_{11} H_{10} \bar{H}_{11} + \bar{H}_4 \bar{H}_1 H_{10} H_{10} \bar{H}_{10} + \bar{H}_4 \bar{H}_1 H_{10} \bar{H}_9 H_9 \\
& + \bar{H}_4 \bar{H}_1 H_{10} \bar{H}_8 H_8 + \bar{H}_4 H_1 H_{10} H_{16} H_{17} + \bar{H}_4 \bar{H}_2 H_2 H_2 \bar{H}_8 + \bar{H}_4 \bar{H}_2 \bar{\Phi}'_{56} \bar{\Phi}_2^{\alpha\beta} H_{10} \\
& + \bar{H}_4 H_2 H_7 \bar{H}_7 \bar{H}_8 + \bar{H}_4 H_2 H_6 \bar{H}_6 \bar{H}_8 + \bar{H}_4 H_2 H_5 \bar{H}_5 \bar{H}_8 + \bar{H}_4 H_2 \Phi_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \bar{H}_8 \\
& + \bar{H}_4 H_2 \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 + \bar{H}_4 H_2 H_{11} \bar{H}_8 \bar{H}_{11} + \bar{H}_4 H_2 H_{10} \bar{H}_8 \bar{H}_{10} + \bar{H}_4 H_2 \bar{H}_9 \bar{H}_8 H_9 \\
& + \bar{H}_4 H_2 \bar{H}_8 \bar{H}_8 H_8 + \bar{H}_1 \bar{H}_1 H_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} + \bar{H}_1 \bar{H}_1 \bar{H}_2 \bar{H}_2 \Phi'_{45} + \bar{H}_1 \bar{H}_1 \bar{\Phi}'_{46} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \\
& + \bar{H}_1 \bar{H}_1 \bar{\Phi}'_{46} \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} + \bar{H}_1 H_1 \Phi_1^{\alpha\beta} H_{11} H_9 + \bar{H}_1 H_1 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + \bar{H}_1 \bar{H}_2 \bar{H}_2 H_2 \bar{\Phi}_2^{\alpha\beta} \\
& + \bar{H}_1 \bar{H}_2 \bar{\Phi}'_{45} \bar{H}_8 \bar{H}_{10} + \bar{H}_1 \bar{H}_2 H_7 \bar{H}_7 \bar{\Phi}_2^{\alpha\beta} + \bar{H}_1 \bar{H}_2 H_6 \bar{H}_6 \bar{\Phi}_2^{\alpha\beta} + \bar{H}_1 \bar{H}_2 H_5 \bar{H}_5 \bar{\Phi}_2^{\alpha\beta} \\
& + \bar{H}_1 \bar{H}_2 \Phi_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} + \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} + \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} + \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} H_{11} \bar{H}_{11} \\
& + \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} H_{10} \bar{H}_{10} + \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} \bar{H}_9 H_9 + \bar{H}_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 H_8 + H_1 H_1 H_2 H_2 \bar{\Phi}'_{45}
\end{aligned}$$

$$\begin{aligned}
& + H_1 H_1 \Phi'_{46} \Phi_1^{\alpha\beta} \Phi_1^{\alpha\beta} + H_1 H_1 \Phi'_{46} \Phi_2^{\alpha\beta} \Phi_2^{\alpha\beta} + H_1 H_1 \Phi'_{46} H_{12} H_{13} + H_1 H_1 \Phi_{45} H_{14} H_{15} \\
& + H_1 \bar{H}_2 \bar{\Phi}_2^{\alpha\beta} H_{16} H_{17} + H_1 H_2 \bar{\Phi}'_{45} H_{10} H_8 + \bar{H}_2 \bar{H}_2 \Phi'_{45} H_{16} H_{17} + \bar{H}_2 \bar{H}_2 \Phi'_{56} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \\
& + \bar{H}_2 \bar{H}_2 \Phi'_{56} \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} + \bar{H}_2 H_2 \Phi_1^{\alpha\beta} H_{11} H_9 + \bar{H}_2 H_2 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + H_2 H_2 \bar{\Phi}'_{56} \Phi_1^{\alpha\beta} \Phi_1^{\alpha\beta} \\
& + H_2 H_2 \bar{\Phi}'_{56} \Phi_2^{\alpha\beta} \Phi_2^{\alpha\beta} + H_2 H_2 \bar{\Phi}'_{56} H_{12} H_{13} + \bar{\Phi}'_{46} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} H_{16} H_{17} + \bar{\Phi}'_{46} \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} H_{16} H_{17} \\
& + \Phi_{46} N_3^c V_9 \bar{H}_9 \bar{H}_9 + \Phi_{46} N_3^c V_8 \bar{H}_8 \bar{H}_8 + \Phi'_{45} N_2^c V_6 \bar{H}_9 \bar{H}_9 + \Phi'_{45} N_2^c V_5 \bar{H}_8 \bar{H}_8 \\
& + \Phi'_{45} \bar{H}_9 \bar{H}_9 \bar{H}_{11} \bar{H}_{11} + \Phi'_{45} \bar{H}_8 \bar{H}_8 \bar{H}_{10} \bar{H}_{10} + \Phi'_{45} H_{11} H_{11} H_9 H_9 + \Phi'_{45} H_{10} H_{10} H_8 H_8 \\
& + \Phi_{45} N_1^c V_3 H_{11} H_{11} + \Phi_{45} N_1^c V_2 H_{10} H_{10} + \Phi_{56} N_3^c V_9 H_{11} H_{11} + \Phi_{56} N_3^c V_8 H_{10} H_{10} \\
& + \Phi_{56} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} H_{14} H_{15} + \Phi_{56} \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} H_{14} H_{15} + N_2^c V_5 \bar{\Phi}_2^{\alpha\beta} H_{10} \bar{H}_8 + N_2^c \bar{\Phi}_2^{\alpha\beta} H_{11} \bar{H}_8 V_{12} \\
& + H_7 \bar{H}_7 \Phi_1^{\alpha\beta} H_{11} H_9 + H_7 \bar{H}_7 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + H_6 \bar{H}_6 \Phi_1^{\alpha\beta} H_{11} H_9 + H_6 \bar{H}_6 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} \\
& + H_5 \bar{H}_5 \Phi_1^{\alpha\beta} H_{11} H_9 + H_5 \bar{H}_5 \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + \Phi_1^{\alpha\beta} \Phi_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} H_{11} H_9 + \Phi_1^{\alpha\beta} \Phi_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} H_{11} H_9 \\
& + \Phi_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + \Phi_1^{\alpha\beta} H_{11} H_{11} \bar{H}_{11} H_9 + \Phi_1^{\alpha\beta} H_{11} H_{10} \bar{H}_{10} H_9 + \Phi_1^{\alpha\beta} H_{11} \bar{H}_9 H_9 H_9 \\
& + \Phi_1^{\alpha\beta} H_{11} \bar{H}_8 H_9 H_8 + \Phi_2^{\alpha\beta} \Phi_2^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} H_{11} H_9 + \Phi_2^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \bar{\Phi}_1^{\alpha\beta} \bar{H}_8 \bar{H}_{10} + \Phi_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_{10} \\
& + \bar{\Phi}_1^{\alpha\beta} H_{11} H_{12} H_9 H_{13} + \bar{\Phi}_2^{\alpha\beta} H_{11} \bar{H}_8 \bar{H}_{11} \bar{H}_{10} + \bar{\Phi}_2^{\alpha\beta} H_{10} \bar{H}_8 \bar{H}_{10} \bar{H}_{10} + \bar{\Phi}_2^{\alpha\beta} \bar{H}_9 \bar{H}_8 \bar{H}_{10} H_9 \\
& + \bar{\Phi}_2^{\alpha\beta} \bar{H}_8 \bar{H}_8 \bar{H}_{10} H_8
\end{aligned} \tag{4.11}$$

5 Flat directions

The model possesses nine local $U(1)$ symmetries, eight in the observable part and one in the hidden part. Six of these are anomalous:

$$\text{Tr} U_1 = \text{Tr} U_2 = -\text{Tr} U_3 = 2\text{Tr} U_4 = -2\text{Tr} U_5 = 2\text{Tr} U_6 = -24. \tag{5.1}$$

$U(1)_L$ and $U(1)_C$ of the $SO(10)$ subgroup are anomaly free. Consequently, the weak hypercharge and the orthogonal combination, $U(1)_{Z'}$, are anomaly free. The hidden sector $U(1)_{H_1}$ is also anomaly free.

Of the six anomalous $U(1)$ s, five can be rotated by an orthogonal transformation to become anomaly free. The unique combination that remains anomalous is: $U_A = k \sum_j [\text{Tr} U(1)_j] U(1)_j$, where j runs over all the anomalous $U(1)$ s and k is a normalisation constant. For convenience, we take $k = \frac{1}{12}$ and therefore the anomalous combination is given by:

$$U_A = -2U_1 - 2U_2 + 2U_3 - U_4 + U_5 - U_6, \quad \text{Tr} Q_A = 180. \tag{5.2}$$

We note that the anomalous $U(1)_A$ combination in (5.2) is similar to that of the model of ref. [3], aside from the changes of the sign of the traces over $U(1)_{1,2}$ and $U(1)_5$.

The five rotated non-anomalous orthogonal combinations are not unique, with different choices related by orthogonal transformations. One choice is given by:

$$U'_1 = U_1 - U_2, \quad U'_2 = U_1 + U_2 + 2U_3, \tag{5.3}$$

$$U'_3 = U_4 + U_5 \quad , \quad U'_4 = U_4 - U_5 - 2U_6, \quad (5.4)$$

$$U'_5 = U_1 + U_2 - U_3 - 2U_4 + 2U_5 - 2U_6. \quad (5.5)$$

Thus, after this rotation there are a total of eight $U(1)$ s free from gauge and gravitational anomalies.

A basis set of (norm-squares of) VEVs of scalar fields satisfying the non-anomalous D -flatness constraints (1.4) can be created en masse [15, 16, 17]. The basis directions can have positive, negative, or zero anomalous charge. In the maximally orthogonal basis used in the singular value decomposition approach of [16, 17], each basis direction is uniquely identified with a particular VEV. That is, although each basis direction generally contains many VEVs, each basis direction contains at least one particular VEV that only appears in it.

A physical D -flat direction D_{phys} , with anomalous charge of sign opposite that of the FI term ξ , is formed from linear combinations of the basis directions,

$$D_{\text{phys}} = \sum_{i=1}^{\# \text{ basis dirs.}} a_i D_i, \quad (5.6)$$

where the integer coefficients a_i are normalised to have no non-trivial common factor.

In our notation, a physical flat direction (5.6) may have a negative norm-square for a vector-like field. This denotes that it is the oppositely charged vector-partner field that acquires the VEV, rather than the field. Basis directions themselves may have vector-like partner directions if all associated fields are vector-like. On the other hand, if in particular, the field generating the VEV uniquely associated with a basis direction does not have a vector-like partner, that basis direction cannot have a vector-like partner direction.

In pursuit of physical all-order flat directions for this model, we first examined directions formed solely from the VEVs of non-Abelian singlet fields. An associated maximally orthogonal basis set, denoted by $\{\mathcal{D}'_{i=1 \text{ to } 13}\}$, containing only non-Abelian singlet VEVs is shown in Table 2.a. The respective unique VEV fields of these basis directions are identified in Table 2.b. Examination of Tables 2.a and 2.b reveals that no physical D -flat directions can be formed solely from VEVs of non-Abelian singlet fields. Since the FI term ξ (1.4) is positive for this model, with $\text{Tr}Q_A = 180$, a physical flat direction must carry a negative anomalous charge. However, of the 13 singlet D -flat basis directions, three carry anomalous charge of $+15$, $+30$, $+30$ while the remaining ten do not carry anomalous charge. Further, the unique VEVed fields for the 3 basis directions with positive anomalous charge do not have corresponding vector-like partner fields. Hence, there are no vector-like paired basis directions with negative anomalous charge. Thus, Tables 2.a and 2.b imply that one or more fields carrying non-Abelian charges must also acquire VEVs in physical D -flat directions. This result is, in itself, not necessarily unexpected, as non-Abelian VEVs have been required for physical (all-order) flat directions in other quasi-realistic free fermionic heterotic models in the past, for example [9].

Thus, we expanded our flat direction search to include VEVs of both non-Abelian singlet fields and non-Abelian charged fields. Our chosen set of 50 maximally orthogonal D -flat basis directions for both non-Abelian singlet VEVs and non-Abelian charged VEVs, denoted by $\{\mathcal{D}_{i=1 \text{ to } 50}\}$, is presented in Table 3.a. The respective unique field VEVs identified with these basis directions are given in Table 3.b. In this enlarged basis the anomalous charges are given in units of $(\frac{Q^{(A)}}{15})$ and the directions containing only singlet VEVs are rotations of those in Table 2.a.

Nine of the 50 directions, denoted $D_{i=1,\dots,9}$, carry one or two units of negative anomalous charge. Twenty basis directions, denoted D_{10} through D_{29} , carry no anomalous charge. Twenty-one basis directions, denoted D_{30} through D_{50} , carry one or two units of positive anomalous charge. All basis directions possessing negative anomalous charge contain $SU(3)_C \otimes SU(2)_L$ charges or hidden sector $SU(4) \otimes \prod_{j=1}^4 SU(2)_j$ charges. (Thus, this basis set also reveals that anomaly cancellation will necessarily break one or more non-Abelian local symmetries.) All of the Φ fields, the $H_{1 \text{ to } 11}$ fields and h have vector-like pairs. Thus, physical flat directions can have negative components for any of these. A subset of these fields, specifically Φ_{46} , Φ'_{45} , $\bar{\Phi}'_{56}$, and $H_{4,5,6,7}$, has VEVs appearing in multiple basis directions. The only non-vector-like field with a VEV that appears in multiple directions is e_3^c .

D_{10} through D_{17} and D_{22} are composed solely of varying combinations of the vector-like fields. Hence, all of these basis directions have corresponding vector-like partner basis directions, $\bar{D}_i \equiv -D_i$, for which the VEV of each field is replaced by the VEV of the vector-like partner field. Thus, in a physical flat direction (5.6), each of the respective integer coefficients a_{10} through a_{17} and a_{22} , may be negative, positive, or zero.

Note that D_7 , D_8 , D_9 and D_{20} are vector-like except for their e_3^c components. Thus, each of a_7 , a_8 , a_9 and a_{20} may be negative, positive, or zero in a physical D -flat direction, so long as the net norm-square VEV of e_3^c is non-negative.³ The remaining basis directions contain at least one unique non-vector-like field VEV. Thus, in a physical flat direction, the coefficients of the remaining basis directions must be non-negative.

What does this mean for a physical D -flat direction formed as a linear combination of the basis directions? For a physical flat direction there are, thus, two specific constraints on the a_i coefficients and one general set of non-negative norm-square constraints on a subset of the a_i . First, negative anomalous charge for a flat direction

³Note that non-vector-like fields, such as e_3^c , that appear in multiple directions with some basis directions having positive and some having negative norm-square components, are common in this process. Further, some models explored in the past have had (at least) one basis direction with two (or more) field VEVs unique to it and with norm-square VEVs with differing signs. This latter type of basis direction can never appear in a physical direction and, hence, implies that the fields unique to it can never appear in a D -flat direction. (If all of the norm-squares of the fields unique to a basis direction were initially negative, then these signs, along with those of the norm-squares of any vector-like field VEVs in that basis direction, could all be changed together to allow the basis direction to appear in a physical direction.)

requires

$$-2 \sum_{i=1}^2 a_i - \sum_{i=3}^9 a_i + \sum_{i=30}^{44} a_i + 2 \sum_{i=45}^{50} a_i < 0. \quad (5.7)$$

Second, a non-negative norm-square VEV for e_3^c requires

$$\begin{aligned} & -6 \sum_{i=1}^2 a_i - 3a_3 - 6 \sum_{i=4}^6 a_i - 2a_7 - 6 \sum_{i=8}^9 a_i - 2 \sum_{i=18}^{19} a_i - a_{20} + a_{21} \\ & -2 \sum_{i=23}^{24} a_i - a_{25} - 2a_{26} + 2a_{27} - 2a_{28} + 2a_{29} + 6a_{30} + 6a_{32} + a_{38} \\ & + 3 \sum_{i=39}^{40} a_i + 6a_{42} + 6 \sum_{i=45}^{47} a_i + 2 \sum_{i=48}^{49} a_i + 6a_{50} \geq 0. \end{aligned} \quad (5.8)$$

Last, for the set of non-vector-like fields that are each identified with a respective unique D -flat direction, the general set of non-negative norm-square VEV constraints is

$$a_i \geq 0 \text{ for } i = 1 \text{ to } 6, 18, 19, 21, 23 \text{ to } 50. \quad (5.9)$$

At low orders, each individual superpotential term also induces several stringent F -term constraints on the a_i coefficients of physical flat directions. As stated prior, the set of constraints from superpotential terms with only singlet fields translate into the requirement that two or more singlet fields in a given superpotential term cannot take on VEVs. For the model under investigation, constraints from third order superpotential terms are especially severe. For this model, all six Φ singlet fields and their vector-like partners appear in third order superpotential terms (specifically, the sixth and seventh lines) of (4.9). Stringent F -flatness from these terms forbids at least 8 of the 12 singlet fields from acquiring VEVs.

For example, when solely third order stringent F -flatness constraints are applied to the six pairs of Φ vector-like singlets (and no F -flatness constraints are applied to the non-Abelian states), there are just nine solution classes that allow the maximum of 4 singlet VEVs. (Flat directions in any of these nine classes are defined by their respective non-Abelian VEVs.)

For three of these nine singlet third order flatness classes, the VEVs are of two fields and their respective vector-like partners: either,

$$\langle \Phi_{45} \rangle, \langle \Phi'_{45} \rangle, \langle \bar{\Phi}_{45} \rangle, \langle \bar{\Phi}'_{45} \rangle \neq 0, \quad \text{or} \quad (5.10)$$

$$\langle \bar{\Phi}_{46} \rangle, \langle \bar{\Phi}'_{46} \rangle, \langle \Phi_{46} \rangle, \langle \Phi'_{46} \rangle \neq 0, \quad \text{or} \quad (5.11)$$

$$\langle \bar{\Phi}'_{56} \rangle, \langle \bar{\Phi}_{56} \rangle, \langle \Phi'_{56} \rangle, \langle \Phi_{56} \rangle \neq 0. \quad (5.12)$$

Higher order stringent flatness constraints can further reduce the allowed number of singlet VEVs of each of these solutions. Further, a component of a D -flat basis

direction in Table 3.a only specifies the difference between the norm-squares of the VEV of a given field and of the given vector-like partner field (if it exists). Completely chargeless VEVs solely involving a field Φ_i and its vector-like partner $\bar{\Phi}_i$ such that $|\langle \Phi_i \rangle|^2 = |\langle \bar{\Phi}_i \rangle|^2$ can always be added to a physical D -flat direction. However, it is preferable for higher order F -flatness to impose that a field and its vector-partner do not simultaneously acquire VEVs. Hence, these three solutions effectively allow only two unique singlet fields to acquire VEVs.

The next three classes of singlet solutions do allow up to four distinct singlet fields to acquire VEVs: either,

$$\langle \Phi_{45} \rangle, \langle \Phi'_{45} \rangle, \langle \Phi_{46} \rangle, \langle \Phi'_{46} \rangle \neq 0, \text{ or,} \quad (5.13)$$

$$\langle \Phi_{45} \rangle, \langle \Phi'_{56} \rangle, \langle \Phi_{56} \rangle, \langle \bar{\Phi}'_{45} \rangle \neq 0, \text{ or,} \quad (5.14)$$

$$\langle \bar{\Phi}_{46} \rangle, \langle \bar{\Phi}_{56} \rangle, \langle \Phi'_{56} \rangle, \langle \Phi'_{46} \rangle \neq 0. \quad (5.15)$$

For the three remaining solution classes, the fields in (5.13), (5.14) and (5.15), are respectively replaced with their vector-like partner fields. For any of these nine stringent F -flat choices, no other Φ singlet fields can acquire VEVs.

Any of the constraints on allowed and disallowed VEVs, such as the above, can be re-expressed in terms of constraints on the a_i coefficients specifying the basis directions contributions to a physical D -flat direction. For example, setting $\langle \Phi_{46} \rangle = 0$ would require

$$\begin{aligned} & 4a_1 + a_2 + 2 \sum_{i=3}^4 a_i + 8a_5 + 2a_6 + a_7 - a_8 - a_9 + a_{10} + a_{16} \\ & - a_{18} + 2a_{19} + a_{20} - a_{21} + a_{23} - 2a_{27} - a_{28} + a_{29} + 4a_{30} \\ & + a_{31} - a_{32} + \sum_{i=33}^{35} a_i - 2 \sum_{i=36}^{37} a_i - 2a_{39} + a_{40} - 2 \sum_{i=41}^{43} a_i + a_{44} \\ & - 4a_{45} + 2a_{46} - a_{47} - 3a_{49} - a_{50} = 0 \quad . \end{aligned} \quad (5.16)$$

To systematically investigate physical D -flat directions with non-Abelian VEVs, over a course of eight months we generated and examined physical D -flat directions composed of from 1 to 6 basis directions. Under the assumption that all VEVs of physical flat directions are nearly of the same order of magnitude, we allowed coefficients of 0 to 20 for the non-vector-like basis directions and coefficients of -20 to 20 for the vector-like basis directions.

To be classified as a physical D -flat direction, a linear combinations of basis directions needed to obey (5.7-5.9) and was, of course, also required to have non-Abelian D -flatness. (The general process by which we enforced non-Abelian D -flatness followed that presented in [16, 17].) Each resulting physical D -flat direction was then tested for stringent F -flatness from all third order through fifth order superpotential terms and additionally for some key sixth order superpotential terms.⁴

⁴While only the third through fifth order superpotential is given in section 4.1, we have generated the complete superpotential to eighth order and can generate it to any required order.

Following the SVD method discussed earlier in section 3 and described in [5, 6, 20], we had planned to then test for possible all-order stringent F -flatness, the subset of physical D -flat directions that had proven stringently F -flat to at least fifth or sixth order. Based on all of the prior models we had investigated, we had expected to find around four to six physical D -flat directions that were, in fact, stringently F -flat to all finite order. However, in contrast we discovered that no physical D -flat directions that we had generated even kept stringent F -flatness through sixth order. So there were no physical D -flat directions to examine for all-order testing! For this model, with its reduced set of singlet fields from the untwisted sector, not even self-cancellation of non-Abelian terms could provide stringent F -flatness through sixth order for any of these physical D -flat directions.

We will continue for several years a search for F -flatness past sixth order for physical D -flat directions in this model that are comprised of seven or more basis directions. After January 2008, the search will be conducted on a 128-node computer system of quad-processors to quicken the pace of the investigation. However, a continued null result is likely: since each of our basis directions contains a unique field VEV, increasing the number of non-zero a_i coefficients linearly increases the minimum number of unique field VEVs. With each increase in number of basis directions composing a physical D -flat direction, the probability of obtaining stringent F -flatness much beyond sixth order further decreases.

6 Conclusions

The string models in the free fermionic formulation gave rise to a large class of quasi-realistic string models, including three generation models that produce solely the MSSM spectrum in the observable Standard Model charged sector of the effective low energy field theory. As such the free fermionic models provides the arena to study how string theory may be related to the observed particle data. In turn, the properties of the models that make them attractive from the point of view of the phenomenological data may be instrumental to uncover unexpected properties of string theory.

In this paper we stumbled upon such a possible novel string feature with regard to supersymmetry breaking. A general expectation from supersymmetric quantum field theories is that if supersymmetry is unbroken at the lowest order in perturbation theory, then it cannot be broken at higher perturbative orders in quantum perturbation theory. The model that we presented in this paper opens up the possibility that string theory may afford other options.

The model is a quasi-realistic three generation string model in the free fermionic formulation that shares many of the characteristics of previous quasi-realistic free fermionic models. It contains three chiral generations charged under the standard-like model subgroup of the underlying $SO(10)$ symmetry of the NAHE-set. The weak-hypercharge possesses the canonical GUT embedding and the model predicts

$\sin^2 \theta_W = 3/8$ at the unification scale. The Higgs spectrum contains one untwisted electroweak doublet pair that couples at the cubic level of the superpotential to the top quark. Like numerous other quasi-realistic string models, the model contains an anomalous $U(1)$ symmetry which generates a Fayet–Iliopoulos D -term. The Fayet–Iliopoulos term would, in general, break supersymmetry, unless there is a direction in the scalar potential which is both D - and F -flat. If such a direction exists it will acquire a VEV, cancel the Fayet–Iliopoulos D -term and restore supersymmetry. The general expectation, due to the corresponding results in supersymmetric point quantum field theories, is that a supersymmetric vacuum does exist due to the fact that string spectrum is Bose–Fermi degenerate and possesses $N = 1$ supersymmetry.

Indeed, in all the previously studied quasi-realistic free fermionic models such supersymmetric flat directions were found. Moreover, all previous models yielded the so called stringent flat directions that can be shown to be exact, *i.e.* flat to all orders of nonrenormalizable terms. The distinct feature of the model discussed in this paper is that it does not admit such stringent flat directions. In this model no physical D -flat direction that we generated kept F -flatness through sixth order. We speculate that only stringent flat directions can be flat to all orders of nonrenormalizable terms. If it is validated, this would indicate that this model, therefore, appears to have no D -flat directions that can be proven to be F -flat to all orders, other than by order by order analysis.

As we discussed this outcome may be a general result of the assignment of boundary conditions to the internal world sheet fermions, which results in the projection of two of the untwisted Higgs pairs. If a non-vanishing F -term does exist, the implication would be that supersymmetry remains unbroken at finite order. The Fayet–Iliopoulos term that breaks supersymmetry is generated at the one-loop level in the perturbative string expansion. On the other hand the string spectrum is Bose–Fermi degenerate and possesses $N = 1$ space–time supersymmetry at the classical level. This would suggest that, contrary to the expectation from supersymmetric point quantum field theories, perturbative supersymmetry breaking in string theory may ensue. Furthermore, the modular invariant one-loop partition function vanishes and, hence, the cosmological constant vanishes at the one loop level as well. The model presented here may therefore represent an example of a quasi-realistic string vacuum with vanishing one-loop cosmological constant and perturbatively broken supersymmetry. Furthermore, the asymmetric boundary conditions of the string model given in eq. (4.1) project all the geometrical moduli of the underlying $Z_2 \times Z_2$ orbifold [34, 35]. Absence of supersymmetric flat solutions would imply that the supersymmetric moduli are fixed as well in this model. Examining the hidden sector gauge group of the model, given in eq. (4.6), we note that it contains $SU(2)^4$ and satisfies the conditions for the dilaton race–track stabilisation mechanism [36]. It remains without saying that many issues still need to be further explored and understood to ascertain the claims of this paper. Nevertheless, the free fermionic models continue to generate intriguing and exciting results that may at the end prove to be relevant

to the elucidation of the connection between string theory and the real world.

Acknowledgements

AEF would like to thank the CERN theory division for hospitality. This work was supported by the STFC, by the University of Liverpool and by Baylor University.

References

- [1] I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos Phys. Lett. B **231** (1989) 65;
J.L. Lopez, D.V. Nanopoulos and K. Yuan, Nucl. Phys. B **399** (1993) 3.
- [2] A.E. Faraggi, D.V. Nanopoulos and K. Yuan, Nucl. Phys. B **335** (1990) 347;
A.E. Faraggi, Phys. Rev. D **46** (1992) 3204.
- [3] A.E. Faraggi, Phys. Lett. B **278** (1992) 131; Nucl. Phys. B **403** (1993) 101; Nucl. Phys. B **407** (1993) 57;
- [4] A.E. Faraggi, Phys. Lett. B **274** (1992) 47; Phys. Rev. D **47** (1993) 5021; Phys. Lett. B **377** (1996) 43; Nucl. Phys. B **487** (1997) 55.
- [5] G.B. Cleaver, A.E. Faraggi and D.V. Nanopoulos, Phys. Lett. B **455** (1999) 135; *Int. J. Mod. Phys. A* **16** (2001) 425;
G.B. Cleaver, A.E. Faraggi, D.V. Nanopoulos and J.W. Walker, Nucl. Phys. B **593** (2001) 471.
- [6] G.B. Cleaver, A.E. Faraggi, D.V. Nanopoulos, and J. Walker, *Mod. Phys. Lett. A* **15** (2000) 1191; G. Cleaver, *Int. J. Mod. Phys. A* **16S1C** (2001) 949.
- [7] A.E. Faraggi, E. Manno and C. Timirgaziu, Eur. Phys. Jour. C **50** (2007) 701.
- [8] I. Antoniadis, G.K. Leontaris and J. Rizos, Phys. Lett. B **245** (1990) 161;
G.K. Leontaris and J. Rizos, Nucl. Phys. B **554** (1999) 3.
- [9] G.B. Cleaver, A.E. Faraggi and C. Savage, Phys. Rev. D **63** (2001) 066001;
G.B. Cleaver, D.J. Clements and A.E. Faraggi, Phys. Rev. D **65** (2002) 106003.
- [10] For review and references, see *e.g.*: A.E. Faraggi, *Int. J. Mod. Phys. A* **19** (2004) 5523; hep-ph/9707311.
- [11] G. B. Cleaver and A. E. Faraggi, *Int. J. Mod. Phys. A* **14**, 2335 (1999) [arXiv:hep-ph/9711339].

- [12] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B **289** (1987) 585;
J. Atick, L.J. Dixon and A. Sen, Nucl. Phys. B **292** (87) 109.
- [13] A. Font, L.E. Ibanez, H.P. Nilles and F. Quevedo, Nucl. Phys. B **307** (1988) 109;
J.A. Casas, E.K. Katehou and C. Munoz, Nucl. Phys. B **317** (1989) 171;
T. Kobayashi and H. Nakano, Nucl. Phys. B **496** (1997) 103;
M.K. Gaillard and J. Giedt, Phys. Lett. B **479** (2000) 308.
- [14] F. Buccella, J.P. Derendinger, S. Ferrara and C.A. Savoy, Phys. Lett. B **115** (1982) 375.
- [15] G.B. Cleaver, M. Cvetič, J. Espinosa, L. Everett, and P. Langacker, Nucl. Phys. B **525** (1998) 3; Nucl. Phys. B **545** (1999) 47.
- [16] G. Cleaver, *Mass Hierarchy and flat directions in string models*, In Miami Beach 1997, Physics of Mass, 1997, 101.
- [17] G.B. Cleaver, A.E. Faraggi, D.V. Nanopoulos and J.W. Walker, Nucl. Phys. B **620** (2002) 259.
- [18] E. Witten, Nucl. Phys. B **202** (1982) 253.
- [19] For a recent review see *e.g.*:
K. Intriligator and N. Seiberg, hep-ph/0702069.
- [20] G. B. Cleaver, arXiv:hep-ph/0703027.
- [21] A.E. Faraggi and E. Halyo, *Int. J. Mod. Phys. A* **11** (1996) 2357.
- [22] A.E. Faraggi and D.V. Nanopoulos, Phys. Rev. D **48** (1993) 3288;
A.E. Faraggi, Nucl. Phys. B **387** (1992) 239; *Int. J. Mod. Phys. A* **14** (1999) 1663.
- [23] H. Kawai, D.C. Lewellen, and S.H.-H. Tye, Nucl. Phys. B **288** (1987) 1;
I. Antoniadis, C. Bachas, and C. Kounnas, Nucl. Phys. B **289** (1987) 87;
I. Antoniadis and C. Bachas, Nucl. Phys. B **289** (1987) 87.
- [24] S. Kalara, J.L. Lopez and D.V. Nanopoulos, Nucl. Phys. B **353** (1991) 650.
- [25] S. Ferrara, L. Girardello, C. Kounnas and M. Porrati, Phys. Lett. B **194** (1987) 368;
S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Phys. Lett. B **194** (1987) 366.
- [26] W. Buchmüller, K. Hamaguchi, O. Lebedev and M. Ratz, arXiv:hep-th/0606187.

- [27] J. Wess and J. Bagger, *Supersymmetry and supergravity*, <http://www.slac.stanford.edu/spires/find/hep/www?irn=5426545SPIRES> entry.
- [28] Press, et al., *Numerical Recipes*, (Cambridge, England: Cambridge University Press, 1989).
- [29] G. B. Cleaver, J. R. Ellis and D. V. Nanopoulos, Nucl. Phys. B **600**, 315 (2001) [arXiv:hep-ph/0009338].
- [30] G. B. Cleaver, A. E. Faraggi and S. Nooij, Nucl. Phys. B **672**, 64 (2003) [arXiv:hep-ph/0301037].
- [31] J. Perkins *et al.*, Phys. Rev. D **75**, 026007 (2007) [arXiv:hep-ph/0510141].
- [32] J. Rizos and K. Tamvakis, Phys. Lett. B **262** (1991) 227.
- [33] A.E. Faraggi, Nucl. Phys. B **428** (1994) 111; Phys. Lett. B **520** (2001) 337.
- [34] A.E. Faraggi, Nucl. Phys. B **728** (2005) 83.
- [35] A.E. Faraggi, Phys. Lett. B **326** (1994) 62; hep-th/9511093;
 J. Ellis, A.E. Faraggi and D.V. Nanopoulos, Phys. Lett. B **419** (1998) 123;
 P. Berglund, J. Ellis, A.E. Faraggi, D.V. Nanopoulos and Z. Qiu, Phys. Lett. B **433** (1998) 269; *Int. J. Mod. Phys. A* **15** (2000) 1345;
 A.E. Faraggi, Phys. Lett. B **544** (2002) 207; hep-th/0411118;
 A.E. Faraggi and R. Donagi, Nucl. Phys. B **694** (2004) 187;
 A.E. Faraggi, S. Förste, M.C. Timirgaziu, JHEP **0608**, (2006) 057.
- [36] N.V. Krasnikov, Phys. Lett. B **193** (1987) 37;
 L.J. Dixon, SLAC-PUB-5229, invited talk given at 15th APS Div. of Particles and Fields General Mtg., Houston, TX, Jan 3-6, 1990. Published in DPF Conf.1990:811-822;
 J.A. Casas, Z. Lalak, C. Munoz and G.G. Ross, Nucl. Phys. B **347** (1990) 243.

F	SEC	$SU(3) \times SU(2)$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(2)_{1,\dots,4} \times SU(4)_{H_1}$	Q_{H_1}
L_1	b_1	$(1, 2)$	$-\frac{3}{2}$	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	$(1, 1, 1, 1, 1)$	0
Q_1		$(3, 2)$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	$(1, 1, 1, 1, 1)$	0
d_1^c		$(\bar{3}, 1)$	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	$(1, 1, 1, 1, 1)$	0
N_1^c		$(1, 1)$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	$(1, 1, 1, 1, 1)$	0
u_1^c		$(\bar{3}, 1)$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	$(1, 1, 1, 1, 1)$	0
e_1^c		$(1, 1)$	$\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	$(1, 1, 1, 1, 1)$	0
L_2	b_2	$(1, 2)$	$-\frac{3}{2}$	0	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$(1, 1, 1, 1, 1)$	0
Q_2		$(3, 2)$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$(1, 1, 1, 1, 1)$	0
d_2^c		$(\bar{3}, 1)$	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$(1, 1, 1, 1, 1)$	0
N_2^c		$(1, 1)$	$\frac{1}{2}$	-1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$(1, 1, 1, 1, 1)$	0
u_2^c		$(\bar{3}, 1)$	$-\frac{1}{2}$	-1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$(1, 1, 1, 1, 1)$	0
e_2^c		$(1, 1)$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	$(1, 1, 1, 1, 1)$	0
L_3	b_3	$(1, 2)$	$-\frac{3}{2}$	0	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	0
Q_3		$(3, 2)$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	0
d_3^c		$(\bar{3}, 1)$	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	0
N_3^c		$(1, 1)$	$\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	0
u_3^c		$(\bar{3}, 1)$	$-\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	0
e_3^c		$(1, 1)$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	0
h	NS	$(1, 2)$	0	-1	0	0	1	0	0	0	$(1, 1, 1, 1, 1)$	0
\bar{h}		$(1, 2)$	0	1	0	0	-1	0	0	0	$(1, 1, 1, 1, 1)$	0
Φ_{56}		$(1, 1)$	0	0	0	0	0	0	1	1	$(1, 1, 1, 1, 1)$	0
$\bar{\Phi}_{56}$		$(1, 1)$	0	0	0	0	0	0	-1	-1	$(1, 1, 1, 1, 1)$	0
Φ'_{56}		$(1, 1)$	0	0	0	0	0	0	1	-1	$(1, 1, 1, 1, 1)$	0
$\bar{\Phi}'_{56}$		$(1, 1)$	0	0	0	0	0	0	-1	1	$(1, 1, 1, 1, 1)$	0
$\bar{\Phi}_{46}$		$(1, 1)$	0	0	0	0	0	-1	0	-1	$(1, 1, 1, 1, 1)$	0
Φ'_{46}		$(1, 1)$	0	0	0	0	0	1	0	-1	$(1, 1, 1, 1, 1)$	0
$\bar{\Phi}'_{46}$		$(1, 1)$	0	0	0	0	0	-1	0	1	$(1, 1, 1, 1, 1)$	0
Φ_{46}		$(1, 1)$	0	0	0	0	0	1	0	1	$(1, 1, 1, 1, 1)$	0
$\xi_{1,2,3}$		$(1, 1)$	0	0	0	0	0	0	0	0	$(1, 1, 1, 1, 1)$	0

Table 1. States with charges.

F	SEC	$SU(3) \times SU(2)$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(2)_{1,\dots,4} \times SU(4)_{H_1}$	Q_{H_1}
Φ_{45}	NS	(1, 1)	0	0	0	0	0	1	1	0	(1, 1, 1, 1, 1)	0
$\bar{\Phi}_{45}$		(1, 1)	0	0	0	0	0	-1	-1	0	(1, 1, 1, 1, 1)	0
Φ'_{45}		(1, 1)	0	0	0	0	0	1	-1	0	(1, 1, 1, 1, 1)	0
$\bar{\Phi}'_{45}$		(1, 1)	0	0	0	0	0	-1	1	0	(1, 1, 1, 1, 1)	0
$\Phi_1^{\alpha\beta}$	$b_1 + b_2$	(1, 1)	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	(1, 2, 1, 2, 1)	0
$\bar{\Phi}_1^{\alpha\beta}$	$\alpha + \beta$	(1, 1)	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	(1, 2, 1, 2, 1)	0
$\Phi_2^{\alpha\beta}$		(1, 1)	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	(2, 1, 2, 1, 1)	0
$\bar{\Phi}_2^{\alpha\beta}$		(1, 1)	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	(2, 1, 2, 1, 1)	0
V_1	$b_1 + 2\gamma$	(1, 1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(1, 1, 1, 1, 6)	0
V_2		(1, 1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1, 1, 1, 1, 1)	-2
V_3		(1, 1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1, 1, 1, 1, 1)	2
V_4	$b_2 + 2\gamma$	(1, 1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	(1, 1, 1, 1, 6)	0
V_5		(1, 1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1, 1, 1, 1, 1)	-2
V_6		(1, 1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1, 1, 1, 1, 1)	2
V_7	$b_3 + 2\gamma$	(1, 1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	(1, 1, 1, 1, 6)	0
V_8		(1, 1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(1, 1, 1, 1, 1)	-2
V_9		(1, 1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(1, 1, 1, 1, 1)	2
V_{10}	$1 + b_2 +$	(1, 1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1, 2, 2, 1, 1)	0
V_{11}	$b_3 + 2\gamma$	(1, 1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(2, 1, 1, 2, 1)	0
V_{12}	$1 + b_1 +$	(1, 1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(2, 1, 1, 2, 1)	0
V_{13}	$b_3 + 2\gamma$	(1, 1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1, 2, 2, 1, 1)	0
V_{14}	$1 + b_1 +$	(1, 1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(2, 1, 1, 2, 1)	0
V_{15}	$b_2 + 2\gamma$	(1, 1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(1, 2, 2, 1, 1)	0
H_1	$b_1 + \alpha$	(1, 2)	0	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	(2, 1, 1, 1, 1)	0
\bar{H}_1		(1, 2)	0	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	(2, 1, 1, 1, 1)	0
H_2	$b_2 + \beta$	(1, 2)	0	0	0	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	(1, 1, 2, 1, 1)	0
\bar{H}_2		(1, 2)	0	0	0	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	(1, 1, 2, 1, 1)	0

Table 1. continued.

F	SEC	$SU(3) \times SU(2)$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(2)_{1,\dots,4} \times SU(4)_{H_1}$	Q_{H_1}
H_3	$b_3 \pm \gamma$	$(3, 1)$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	1
H_4		$(1, 2)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	1
H_5		$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	1
H_6		$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	1
H_7		$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 1)$	1
\bar{H}_3		$(3, 1)$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	-1
\bar{H}_4		$(1, 2)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	-1
\bar{H}_5		$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	-1
\bar{H}_6		$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	-1
\bar{H}_7		$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$	0	0	$\frac{1}{2}$	$(1, 1, 1, 1, 1)$	-1
H_8	$b_2 + b_3$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	$(1, 1, 2, 1, 1)$	-1
\bar{H}_8	$\beta \pm \gamma$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	0	$(1, 1, 2, 1, 1)$	1
H_9	$1 + b_1$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	$(1, 2, 1, 1, 1)$	1
\bar{H}_9	$+\beta \pm \gamma$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	0	$(1, 2, 1, 1, 1)$	-1
H_{10}	$b_1 + b_3$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	0	$(2, 1, 1, 1, 1)$	1
\bar{H}_{10}	$\alpha \pm \gamma$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	0	$(2, 1, 1, 1, 1)$	-1
H_{11}	$1 + b_2 +$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	0	$(1, 1, 1, 2, 1)$	-1
\bar{H}_{11}	$+\alpha \pm \gamma$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	0	$(1, 1, 1, 2, 1)$	1
H_{12}	$1 + b_3 + \alpha$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$(1, 1, 1, 1, 4)$	0
H_{13}	$+\beta \pm \gamma$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$(1, 1, 1, 1, \bar{4})$	0
H_{14}	$1 + b_2 + \alpha$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1, 1, 1, 1, 4)$	0
H_{15}	$+\beta \pm \gamma$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1, 1, 1, 1, \bar{4})$	0
H_{16}	$1 + b_1 + \alpha$	$(1, 1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$(1, 1, 1, 1, 4)$	0
H_{17}	$+\beta \pm \gamma$	$(1, 1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$(1, 1, 1, 1, \bar{4})$	0

Table 1. continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$	Φ'_{45} e_1^c	$\bar{\Phi}'_{56}$ e_2^c	V_3 e_3^c	V_2 H_7	V_6 H_6	V_5 H_5	V_9	V_8	N_1^c	N_2^c	N_3^c	Φ'_{46}	Φ_{45}
\mathcal{D}'_1	1	1 0	-2 0	1 0	0 3	0 2	0 2	0 2	0	3	0	0	0	0	0
\mathcal{D}'_2	2	2 0	-1 0	-1 0	0 6	6 1	0 4	0 7	0	0	0	0	0	0	0
\mathcal{D}'_3	2	-1 0	-1 0	2 0	0 6	0 1	0 7	6 4	0	0	0	0	0	0	0
\mathcal{D}'_4	0	1 0	-1 0	0 2	0 -2	0 1	0 -1	0 0	0	0	0	0	0	0	0
\mathcal{D}'_5	0	-1 0	1 0	0 0	0 0	0 1	0 -1	0 0	0	0	0	2	-2	0	0
\mathcal{D}'_6	0	0 0	-1 2	1 0	0 -2	0 1	0 0	0 -1	0	0	0	0	0	0	0
\mathcal{D}'_7	0	1 0	0 0	-1 0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	1
\mathcal{D}'_8	0	1 1	-1 0	0 0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0
\mathcal{D}'_9	0	0 0	-1 0	1 0	0 0	0 0	0 0	0 0	0	0	0	0	0	1	0
\mathcal{D}'_{10}	0	0 0	0 0	0 0	0 -1	0 0	0 -1	0 -1	1	0	0	0	0	0	0
\mathcal{D}'_{11}	0	0 0	1 0	-1 0	0 0	0 1	0 0	0 -1	0	0	2	0	-2	0	0
\mathcal{D}'_{12}	0	-1 0	1 0	0 0	0 -2	0 -1	2 -1	0 -2	0	0	0	0	0	0	0
\mathcal{D}'_{13}	0	0 0	1 0	-1 0	2 -2	0 -1	0 -2	0 -1	0	0	0	0	0	0	0

Table 2.a. D -Flat direction basis of non-Abelian singlet fields.

Column 2 specifies the anomalous charge and columns 3 through 16 specify the norm-square VEV components of each basis direction. The six fields e_i^c and $H_{5,6,7}$ carry hypercharge, the remaining do not. A negative component indicates the vector partner of a field (if it exists) must take on VEV rather than the field.

FD	VEV
\mathcal{D}'_1	V_8
\mathcal{D}'_2	V_2
\mathcal{D}'_3	V_5
\mathcal{D}'_4	e_2^c
\mathcal{D}'_5	N_2^c
\mathcal{D}'_6	e_1^c
\mathcal{D}'_7	Φ_{45}
\mathcal{D}'_8	$\bar{\Phi}_{56}$
\mathcal{D}'_9	Φ'_{46}
\mathcal{D}'_{10}	V_9
\mathcal{D}'_{11}	N_1^c
\mathcal{D}'_{12}	V_6
\mathcal{D}'_{13}	V_3

Table 2.b. Unique VEV associated with each non-Abelian singlet field D -Flat basis direction.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} V_1 u_2^c	V_8 V_4 V_7 u_3^c H_3	N_1^c N_2^c N_3^c h	N_2^c N_3^c L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_1	-2	4	1	1	0	0	0	0	0	0	6	0	0	0
		0	0	0	-6	-1	-4	-7						
		0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0
		12	0	0										
\mathcal{D}_2	-2	1	1	4	0	0	0	0	0	0	0	6	0	0
		0	0	0	-6	-1	-7	-4						
		0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0
		12	0	0										
\mathcal{D}_3	-1	2	-1	2	0	0	0	0	0	0	0	0	3	0
		0	0	0	-3	-2	-2	-2						
		0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0
		6	0	0										
\mathcal{D}_4	-1	2	-1	8	0	0	0	0	0	0	0	0	0	0
		0	0	0	-6	4	1	-5						
		0	0	0	0	0	12	0	0	0	0	0	0	0
		0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0										
\mathcal{D}_5	-1	8	-1	2	0	0	0	0	0	0	0	0	0	0
		0	0	0	-6	4	-5	1						
		0	0	0	0	0	0	12	0	0	0	0	0	0
		0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0										
\mathcal{D}_6	-1	2	5	2	0	0	0	0	0	0	0	0	0	0
		0	0	0	-6	-2	1	1						
		0	0	0	0	0	0	0	12	0	0	0	0	0
		0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0										

Table 3.a D -Flat direction basis of all fields.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_7	-1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	-2	-1	-2	-2							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	3	0	0	0	0
		2	0	0											
\mathcal{D}_8	-1	-1	-1	2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	-6	-2	-5	-5							
		0	0	0	-6	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		6	0	0											
\mathcal{D}_9	-1	-1	2	2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	-6	-2	-5	-5							
		0	-6	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		6	0	0											
\mathcal{D}_{10}	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	1
		0	0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{11}	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	2											
\mathcal{D}_{12}	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	2	0											

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{13}	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0							
		0	0	2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		-2	0	0											
\mathcal{D}_{14}	0	0	-1	1	0	0	0	0	0	0	0	0	0	1	0
		0	0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{15}	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0							
		2	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{16}	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
		1	0	0	0	0	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{17}	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0							
		0	0	0	0	2	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		-2	0	0											

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} V_1 u_2^c	V_8 V_4 V_7 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{18}	0	-1	1	0	0	0	2	0	0	0	0	0	0	0	0
		0	0	0	-2	-1	-1	-2							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{19}	0	2	1	-1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	-2	1	0	-3							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		6	0	0	0	0	0	0	0	0	0	0	0	0	0
		2	0	0											
\mathcal{D}_{20}	0	1	-1	1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	-1	-1	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	3	0	0	0	0	0	0	0	0	0	0	0
		1	0	0											
\mathcal{D}_{21}	0	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	1	-2	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	3	0	0	0	0	0
		2	0	0											
\mathcal{D}_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	-1	0	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	1	0	0	0
		1	0	0											
\mathcal{D}_{23}	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	2	-2	1	-1	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{24}	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0
		0	2	0	-2	1	0	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathcal{D}_{25}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
		0	0	0	-1	0	-1	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathcal{D}_{26}	0	0	1	-1	2	0	0	0	0	0	0	0	0	0	0
		0	0	0	-2	-1	-2	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathcal{D}_{27}	0	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	2	-1	0	-3							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	6	0	0	0	0	0	0	0
\mathcal{D}_{28}	0	4	0	0											
		-1	1	2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	-2	1	-3	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
\mathcal{D}_{29}	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0
		2	0	0											
		1	-1	-2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	2	-1	-3	0							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	6	0	0	0	0	0	0
		4	0	0											

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{30}	1	4	1	-2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	6	-4	5	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	12	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{31}	1	1	-2	-2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	-1	2	-1							
		0	0	0	0	0	0	0	0	0	6	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{32}	1	-2	7	-2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	6	2	-1	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	12
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{33}	1	1	1	-2	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	-1	-1	2							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	6	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{34}	1	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	2	-1	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		6	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											
\mathcal{D}_{35}	1	1	-2	1	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	2	-1	-1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	6	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0											

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{36}	1	-2 0 0 0 0 0	1 0 0 0 0 0	1 0 0 6 0 0	0 0 0 0 0 0	0 -1 0 0 0 0	0 2 0 0 0 0	0 -1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{37}	1	-2 0 0 0 0 0	1 0 0 0 0 0	1 0 0 6 0 0	0 0 0 0 0 0	0 -1 0 0 0 0	0 2 0 0 0 0	0 -1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{38}	1	0 0 0 0 0 -4	-1 0 0 0 0 0	0 0 0 0 0 0	0 1 0 0 0 0	0 0 0 0 0 3	0 2 0 0 0 0	0 2 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{39}	1	-2 0 0 0 0 -3	1 0 0 0 0 0	-2 0 0 0 0 0	0 3 0 0 0 0	0 -1 0 0 0 0	0 2 0 0 0 0	0 2 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 3
\mathcal{D}_{40}	1	1 0 0 0 0 0	-2 0 0 0 0 0	1 0 0 0 0 0	0 3 0 0 0 0	0 2 0 0 0 0	0 2 0 0 0 0	0 2 0 0 0 0	0 0 0 0 0 0	3 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{41}	1	-2 0 0 0 0 0	-2 0 0 0 0 0	1 0 0 0 0 0	0 0 0 0 0 0	0 -1 0 0 0 0	0 -1 0 0 0 0	0 2 0 0 0 0	0 0 0 0 0 0	0 6 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{42}	1	-2 0 0 0 0 0	1 0 0 0 0 0	4 0 0 0 0 0	0 6 0 0 0 0	0 -4 0 0 0 0	0 -1 0 0 0 0	0 5 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 12 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{43}	1	-2 0 0 0 0 0	1 0 0 0 0 0	-2 0 0 0 0 0	0 0 0 0 0 0	0 2 0 0 0 0	0 -1 0 0 0 0	0 -1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 6 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{44}	1	1 0 0 0 0 0	1 0 0 0 0 0	-2 0 0 0 0 0	0 0 0 0 0 0	0 -1 0 0 0 0	0 -1 0 6 0 0	0 2 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{45}	2	-4 0 0 0 0 -6	-1 0 0 0 0 0	-1 0 0 0 0 0	0 6 0 0 0 0	0 1 0 0 0 0	0 4 0 0 0 0	0 1 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 6	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{46}	2	2 0 0 0 0 0	-1 0 0 0 0 0	-1 0 0 0 0 0	0 6 0 0 0 0	6 1 0 0 0 0	0 4 0 0 0 0	0 7 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
\mathcal{D}_{47}	2	-1 0 0 0 0 0	-1 0 0 0 0 0	2 0 0 0 0 0	0 6 0 0 0 0	0 1 0 0 0 0	0 7 0 0 0 0	6 4 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0

Table 3.a continued.

FD	$\frac{Q^{(A)}}{15}$	Φ_{46} $\bar{\Phi}_{56}$ $\bar{\Phi}_{1,2}^{\alpha\beta}$ V_{15} Q_1 H_4	Φ'_{45} e_1^c H_{11} V_{14} Q_2 \bar{H}_1	$\bar{\Phi}'_{56}$ e_2^c H_{10} V_{13} Q_3 \bar{H}_2	V_3 e_3^c \bar{H}_9 V_{12} d_1^c	V_2 H_7 \bar{H}_8 V_{10} d_2^c	V_6 H_6 H_{16} V_{11} d_3^c	V_5 H_5 H_{14} u_1^c	V_9 H_{12} u_2^c	V_8 V_1 u_3^c	N_1^c V_4 H_3	N_2^c V_7 h	N_3^c H_{17} L_1	Φ'_{46} H_{15} L_2	Φ_{45} H_{13} L_3
\mathcal{D}_{48}	2	0	1	-3	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	2	3	4	1							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	6	0	0	0	0	0	0	0	0	0	0
		-8	0	0											
\mathcal{D}_{49}	2	-3	1	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	2	3	1	4							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	6	0	0	0	0	0	0	0	0	0
		-8	0	0											
\mathcal{D}_{50}	2	-1	-1	-4	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	6	1	1	4							
		0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0								
		0	0	0	0	0	0	0	0	0	0	0	0	6	0
		-6	0	0											

Table 3.a continued.

FD	VEV	FD	VEV	FD	VEV	FD	VEV	FD	VEV
\mathcal{D}_1	N_1^c	\mathcal{D}_{11}	H_2	\mathcal{D}_{21}	u_3^c	\mathcal{D}_{31}	V_4	\mathcal{D}_{41}	V_1
\mathcal{D}_2	N_2^c	\mathcal{D}_{12}	\bar{H}_1	\mathcal{D}_{22}	h	\mathcal{D}_{32}	H_{13}	\mathcal{D}_{42}	H_{17}
\mathcal{D}_3	N_3^c	\mathcal{D}_{13}	H_{10}	\mathcal{D}_{23}	e_2^c	\mathcal{D}_{33}	V_{10}	\mathcal{D}_{43}	V_7
\mathcal{D}_4	H_{16}	\mathcal{D}_{14}	Φ'_{46}	\mathcal{D}_{24}	e_1^c	\mathcal{D}_{34}	V_{15}	\mathcal{D}_{44}	V_{11}
\mathcal{D}_5	H_{14}	\mathcal{D}_{15}	$\bar{\Phi}_{56}$	\mathcal{D}_{25}	V_9	\mathcal{D}_{35}	V_{14}	\mathcal{D}_{45}	L_1
\mathcal{D}_6	H_{12}	\mathcal{D}_{16}	$\bar{\Phi}_{1,2}^{\alpha\beta}$	\mathcal{D}_{26}	V_3	\mathcal{D}_{36}	V_{13}	\mathcal{D}_{46}	V_2
\mathcal{D}_7	H_3	\mathcal{D}_{17}	\bar{H}_8	\mathcal{D}_{27}	u_1^c	\mathcal{D}_{37}	V_{12}	\mathcal{D}_{47}	V_5
\mathcal{D}_8	\bar{H}_9	\mathcal{D}_{18}	V_6	\mathcal{D}_{28}	Q_2	\mathcal{D}_{38}	d_3^c	\mathcal{D}_{48}	d_1^c
\mathcal{D}_9	H_{11}	\mathcal{D}_{19}	Q_1	\mathcal{D}_{29}	u_2^c	\mathcal{D}_{39}	L_3	\mathcal{D}_{49}	d_2^c
\mathcal{D}_{10}	Φ_{45}	\mathcal{D}_{20}	Q_3	\mathcal{D}_{30}	H_{15}	\mathcal{D}_{40}	V_8	\mathcal{D}_{50}	L_2

Table 3.b. Unique VEV associated with each D -Flat basis direction.